Additional Assessment Materials

Summer 2021

Pearson Edexcel GCE in As Mathematics

8MA0_01 (Public release version)

Resource Set 1: Topic 6

Exponential and logs

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Additional Assessment Materials, Summer 2021

General guidance to Additional Assessment Materials for use in 2021

Context

- Additional Assessment Materials are being produced for GCSE, AS and A levels (with the exception of Art and Design).
- The Additional Assessment Materials presented in this booklet are an optional part of the range of evidence teachers may use when deciding on a candidate's grade.
- 2021 Additional Assessment Materials have been drawn from previous examination materials, namely past papers.
- Additional Assessment Materials have come from past papers both published (those materials available publicly) and unpublished (those currently under padlock to our centres) presented in a different format to allow teachers to adapt them for use with candidate.

Purpose

- The purpose of this resource to provide qualification-specific sets/groups of questions covering the knowledge, skills and understanding relevant to this Pearson qualification.
- This document should be used in conjunction with the mapping guidance which will map content and/or skills covered within each set of questions.
- These materials are only intended to support the summer 2021 series.

1.

A student's attempt to solve the equation $2\log_2 x - \log_2 \sqrt{x} = 3$ is shown below.

$2\log_2 x - \log_2 \sqrt{x} = 3$	
$2\log_2 \left(\frac{x}{\sqrt{x}}\right) = 3$	using the subtraction law for logs
$2\log_2 \left(\sqrt{x}\right) = 3$	simplifying
$\log_2 x = 3$	using the power law for logs
$x = 3^2 = 9$	using the definition of a log

(a) Identify two errors made by this student, giving a brief explanation of each.

(2)

① a) 1. First should have changed $2\log_2 x$ into $\log_2 x^2$ before simplifying the logarithms.

2. $\log_2 x = 3 \neq 3^2 = x$. The correct log law equals $2^3 = x = 8$.

(b) Write out the correct solution.

(3)

b) $2\log_2 x - \log_2 \sqrt{x} = 3$

$\log_2 x^2 - \log_2 x^{\frac{1}{2}} = 3$

$\Rightarrow \log_2 x^{3/2} = 3$

$2^3 = x^{3/2}$

$8 = x^{3/2}$

$\boxed{\Rightarrow x = 4}$

(Total for Question 1 is 6 marks)

---

2. A student was asked to give the exact solution to the equation

$$2^{2x+4} - 9(2^x) = 0$$

The student's attempt is shown below:

$2^{2x+4} - 9(2^x) = 0$

$2^{2x} + 2^4 - 9(2^x) = 0$

Let $2^x = y$

$y^2 - 9y + 8 = 0$

$(y - 8)(y - 1) = 0$

$y = 8$ or $y = 1$

So $x = 3$ or $x = 0$

(a) Identify the two errors made by the student.

(2)

② a) Error 1:

$2^{2x+4} \neq 2^{2x} + 2^4$.

The correct method would be $2^{2x} \times 2^4$.

Error 2:

$2^4 \neq 8$. $2^4 = 16$

(b) Find the exact solution to the equation.

(2)

b) $2^{2x+4} - 9(2^x) = 0$

$2^{2x} \times 2^4 - 9(2^x) = 0$

$(2^x)^2 \times 16 - 9(2^x) = 0$

$16(2^x)^2 - 9(2^x) = 0$

$\boxed{\text{let } y = 2^x}$

$16y^2 - 9y = 0$

$y(16y - 9) = 0$

$y = 0$ or $16y - 9 = 0$

$y = \frac{9}{16}$

$2^x = 0$ or $2^x = \frac{9}{16}$

$\cancel{\log_2 0} = x$

no solutions

$\log_2\left(\frac{9}{16}\right) = x$

$\Rightarrow x = \log_2\left(\frac{9}{16}\right)$

(Total for Question 2 is 4 marks)

3.

Find any real values of x such that

$$2\log_4(2 - x) - \log_4(x + 5) = 1$$

(6)

3) $2\log_4(2-x) - \log_4(x+5) = 1$

$\Rightarrow \log_4(2-x)^2 - \log_4(x+5) = 1$

$\Rightarrow \log_4\left(\frac{(2-x)^2}{x+5}\right) = 1$

$\Rightarrow 4 = \frac{(2-x)(2-x)}{(x+5)}$

$\Rightarrow 4 = \frac{4 - 4x + x^2}{(x+5)}$

$\Rightarrow 4x + 20 = 4 - 4x + x^2$

$\Rightarrow x^2 - 8x - 16 = 0$

$x = 4 + 4\sqrt{2}$ or $x = 4 - 4\sqrt{2}$

Since we need $2 - x > 0$ (as log function undefined otherwise), we have the solution:

$\boxed{x = 4 - 4\sqrt{2}}$

(Total for Question 3 is 6 marks)

4.

Figure 3

The value of a rare painting, £V, is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10} V$ since 1st January 1980.

The equation of line l is $\log_{10} V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q.

(4)

④ a) $V = pq^t$

$\log_{10} V = \log_{10} pq^t$

$\Rightarrow \log_{10} V = \log_{10} p + \underline{t \log_{10} q}$ gradient

y intercept

$\log_{10} V = 0.05t + 4.8$

$\therefore 0.05 = \log_{10} q$

$\Rightarrow q = 10^{0.05}$

$q = 1.122$

$\log_{10} p = 4.8$

$\Rightarrow p = 10^{4.8}$

$\Rightarrow p = 6310$

(b) With reference to the model interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

b) i) $p = 6310$ = the value, £V, of the painting at $t = 0$, 1st January 1980. hence £6310 is the value of the painting when it was first recorded.

ii) q, 1.122, is the rate of increase in the value of the painting in £ year^{-1}.

4.

(c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds.

(2)

c) Find £V when $t=30$.

$V = 6310\,(1.122)^{t}$

$\Rightarrow V = 6310\,(1.122)^{30}$

$\Rightarrow V = 199441.2865$

$\therefore V =$ £200 000 on 1st January 2010.

(Total for Question 4 is 8 marks)

5.

The growth of pond weed on the surface of a pond is being investigated.

The surface area of the pond covered by the weed, A m^2, can be modelled by the equation

$$A = 0.2e^{0.3t}$$

where t is the number of days after the start of the investigation.

(a) State the surface area of the pond covered by the weed at the start of the investigation.
(1)

$A = 0.2e^{0.3t}$

a) when $t = 0$, $A = 0.2\ \text{m}^2$

(b) Find the rate of increase of the surface area of the pond covered by the weed, in m^2/day, exactly 5 days after the start of the investigation.
(2)

b) when $t = 5$

$$\frac{dA}{dt} = 0.6e^{0.3t}$$
$$= 0.6e^{0.3(5)}$$
$$= 2.69\ \text{m}^2\ \text{day}^{-1}$$

Given that the pond has a surface area of 100 m^2,

(c) find, to the nearest hour, the time taken, according to the model, for the surface of the pond to be fully covered by the weed.
(4)

c) $100 = 0.2e^{0.3t}$

$500 = e^{0.3t}$

$\ln(500) = 0.3t$

$t = \dfrac{\ln(500)}{0.3}$

$\Rightarrow t = 20.7$ days

$\therefore$ $t = 497$ hours or 20 days and 17 hours

The pond is observed for one month and by the end of the month 90% of the surface area of the pond was covered by the weed.

(d) Evaluate the model in light of this information, giving a reason for your answer.
(1)

d) The model is inaccurate because it predicted the entire surface of the pond would be covered in 20.7 days i.e. less than one month, however after one month only 90% was covered, so the model overstimated the rate of growth of the weed.

(Total for Question 5 is 8 marks)

6.

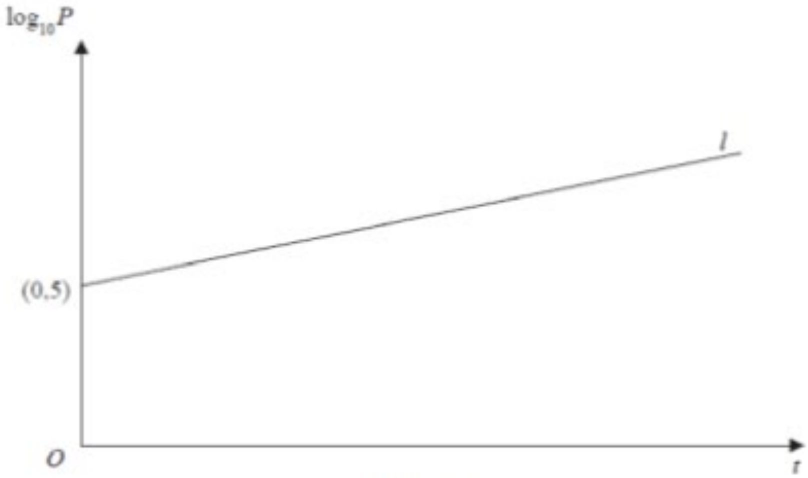

Figure 2

A town's population, P, is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded.

The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10}P$ for the population over a period of 100 years.

The line l meets the vertical axis at (0, 5) as shown. The gradient of l is $\frac{1}{200}$.

(a) Write down an equation for l.

(2)

6 a) $l = 5 + \frac{1}{200}t$

(b) Find the value of a and the value of b.

(4)

b) $P = ab^t$

$\log_{10} P = \log_{10} ab^t$

$\Rightarrow \log_{10} P = \log_{10} a + t\log_{10} b$

$\therefore \log_{10} a = 5$ and $\log_{10} b = \frac{1}{200}$

$\Rightarrow a = 10^5$ $\Rightarrow b = 10^{1/200}$

$a = 1 \times 10^5$ $\Rightarrow b = 1.0116$

(c) With reference to the model interpret

(i) the value of the constant a,

(ii) the value of the constant b.

(2)

c) i) a, 1×10^5, is the town's population at $t=0$ years; i.e. when the population was first recorded.

ii) b, 1.0116, is the rate of increase of the town's population over 100 years

(d) Find

(i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

d) i) when $t = 100$:

$$P = ab^t$$

$$\Rightarrow P = (1 \times 10^5)(1.0116)^t$$

$$P = (1 \times 10^5)(1.0116)^{100}$$

$$\therefore P = 316\,870.6883$$

hence $P = 317000$

ii) $200\,000 = (1 \times 10^5)(1.0116)^t$

$$(1.0116)^t = 2$$

$$(t)\ln(1.0116) = \ln 2$$

$$t = \frac{\ln 2}{\ln(1.0116)}$$

hence $t = 60.1$

It takes 60.1 years for the town's population to reach 200,000.

(e) State two reasons why this may not be a realistic population model.

(2)

e) 1. The model doesn't take into account deaths per year

2. The population size cannot continue increasing indefinitely as the town has a size limit i.e. there eventually won't be enough room for all the people.

(Total for Question 6 is 13 marks)

7.

The temperature, θ°C, of a cup of tea t minutes after it was placed on a table in a room, is modelled by the equation

$$\theta = 18 + 65e^{-\frac{t}{8}} \qquad t \geqslant 0$$

Find, according to the model,

(a) the temperature of the cup of tea when it was placed on the table,

(1)

$\theta = 18 + 65 = 83°$

(b) the value of t, to one decimal place, when the temperature of the cup of tea was 35 °C.

(3)

b) $35 = 18 + 65e^{-\frac{t}{8}}$

$\frac{17}{65} = e^{-\frac{t}{8}}$

$\Rightarrow \ln e^{-t/8} = \ln\left(\frac{17}{65}\right)$

$\Rightarrow -\frac{t}{8} = \ln\left(\frac{17}{65}\right)$

$\Rightarrow \boxed{t = 10.7}$

(c) Explain why, according to this model, the temperature of the cup of tea could not fall to 15 °C.

(1)

Due to the positivity of the exponential, according to the model the temperature cannot fall below 18°C.

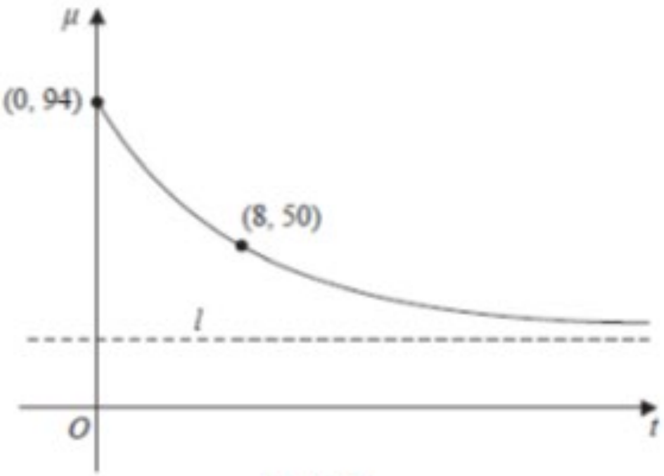

Figure 2

The temperature, μ°C, of a second cup of tea t minutes after it was placed on a table in a different room, is modelled by the equation

$$\mu = A + Be^{-\frac{t}{8}} \qquad t \geqslant 0$$

$$\mu = A + Be^{-t/8}$$

where A and B are constants.

Figure 2 shows a sketch of μ against t with two data points that lie on the curve.

The line l, also shown on Figure 2, is the asymptote to the curve.

Using the equation of this model and the information given in Figure 2

(d) find an equation for the asymptote l.

(4)

line l is $y = A$

d) $94 = A + B \Rightarrow A = 94 - B$

$50 = A + Be^{-1}$

$\Rightarrow 50 = (94 - B) + \frac{B}{e}$

$\Rightarrow -44 = -B + \frac{B}{e}$

$\Rightarrow 44e = Be - B$

$\Rightarrow 44e = B(e-1)$

$\therefore B = \frac{44e}{e-1}$ and $A = 94 - \frac{44e}{e-1}$

hence line $l \Rightarrow$ $\boxed{y = 94 - \frac{44e}{e-1}}$

(Total for Question 7 is 8 marks)